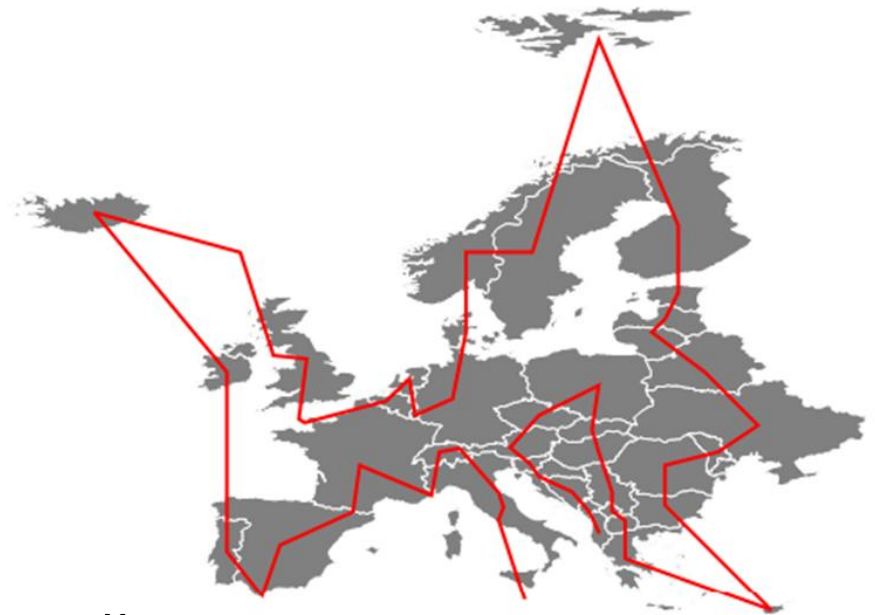


Approximation Algorithms



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Analysis of Algorithms and Heuristic Problem Solving
Edition 2025

Contents

- performance ratios
 - examples of approximation algorithms
 - non-existence of approximation algorithms
-
- Literature:
Cormen et al.: Introduction to algorithms, 2009/2022, Chapter 35
 - to better understand the context of approximation algorithms,.
refresh your knowledge about NP-completeness in Cormen et al.:
Introduction to algorithms, 2009, Chapter 34

Performance ratios

- approximation ratio is a ratio between the cost of approximate and optimal solution of a problem
- see Cormen et al: Introduction to algorithms, 2009/2022, Chapter 35

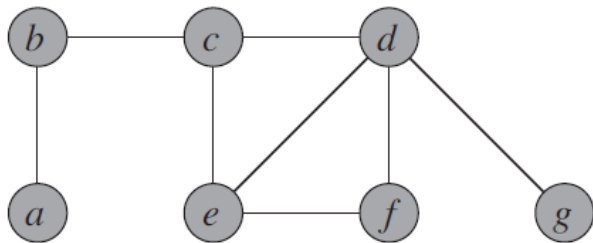
Vertex-cover

Vertex cover of a graph $G=(V, E)$ is a set of vertices that cover all edges of the graph.

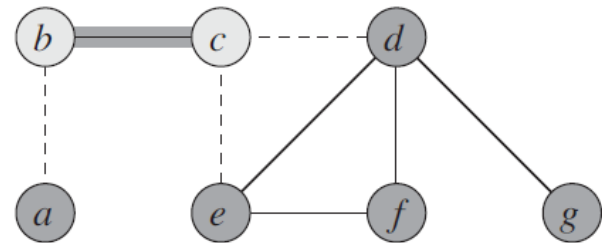
APPROX-VERTEX-COVER (G)

```
1   $C = \emptyset$ 
2   $E' = G.E$ 
3  while  $E' \neq \emptyset$ 
4      let  $(u, v)$  be an arbitrary edge of  $E'$ 
5       $C = C \cup \{u, v\}$ 
6      remove from  $E'$  every edge incident on either  $u$  or  $v$ 
7  return  $C$ 
```

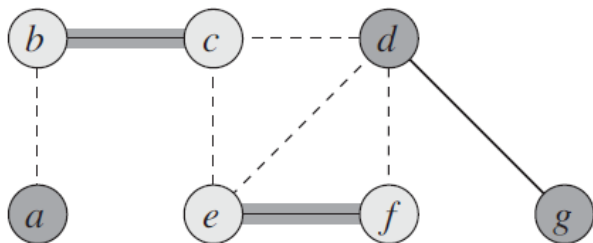
Illustration of Approx-Vertex-Cover algorithm



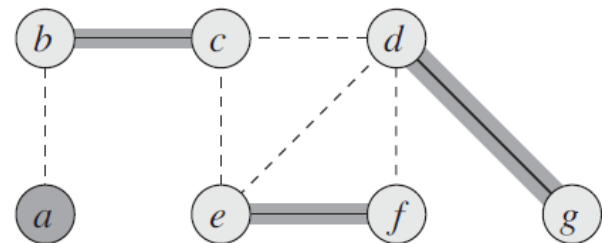
(a)



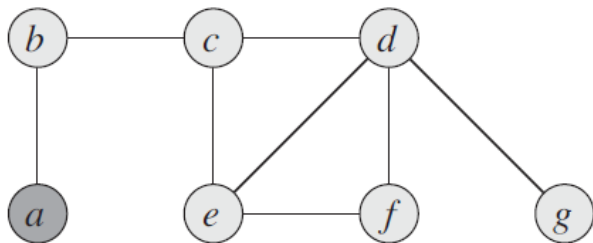
(b)



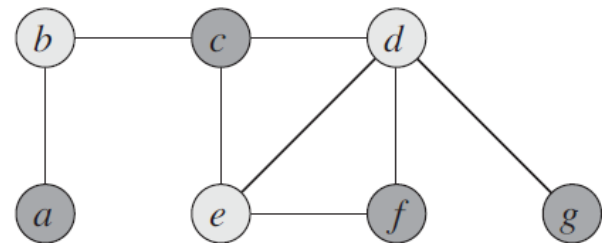
(c)



(d)



(e)



(f)

General TSP

- Non-existence of approximation algorithm for general TSP

MAX-3CNF-SAT

- expected approximation ratio is an expected ratio between the cost of approximate and optimal solution of a problem of a randomized algorithm
- randomized algorithm: randomly assign each of the variables with 0 or 1 with probability 0.5
- this algorithm is $8/7$ -approximation algorithm